HOMEWORK FOR 3RD. EXAM

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(1) Consider the following context-free grammar G:

$$S \to ABS \mid AB$$
$$A \to aA \mid a$$
$$B \to bA$$

Which of the following strins are in $L(G)$ and which are not? Provide derivations for those that are in $L(G)$ and reasons for those that are not.
(a) $aabaab$
(b) $aaaaba$
(c) $aabbaa$
(d) $abaaba$

(2) Give a context-free grammar for the set $\mathtt{PAREN}_2$ of balanced strings of parentheses of two types `( )` and `[ ]`. For example, `([()[]]([])` $\in \mathtt{PAREN}_2$, but $[(]) \notin \mathtt{PAREN}_2$. Use the following inductive definition:$\mathtt{PAREN}_2$ is the smallest set of strings such that
(a) $\epsilon \in \mathtt{PAREN}_2$;
(b) if $x \in \mathtt{PAREN}_2$, then so are (x) and $[x]$;
(c) if x and y are both in $\mathtt{PAREN}_2$, then so is xy.
(*Hint:* Your grammar should closely model the inductive definition of the set.)

(3) Give a grammar with no $\epsilon-$ or unit productions generating the set $L(G) - \{\epsilon\}$, where G is the grammar

$$S \to aSbb \mid T$$
$$T \to bTaa \mid S \mid \epsilon$$

(4) Give grammars in Chomsky normal form for the following context-free languages.
(a) $\{a^n b^{2n} c^k \mid k, n \geq 1\}$
(b) $\{a^n b^k a^n \mid k, n \geq 1\}$

(5) Prove that the set

$$\mathtt{PRIMES} = \{a^p \mid p \text{ is prime}\}$$

is not context-free. (*Hint*: $\forall\ k$ integer $\exists p$ prime such that $p \geq k$).

(6) Is the following language regular, context-free or not context-free? Give a justification.

$$\{a^{2^n} \mid n \geq 0\}$$

References

[1] Dexter C. Kozen, Automata Theory and Computability, Springer, 1997.