*(b) (Stanat and Weiss [117]) A set $A \subseteq \Sigma^*$ is regular if and only if there exists $k \geq 0$ such that for all $y \in \Sigma^*$ with $|y| \geq k$, there exist $u, v, w \in \Sigma^*$ such that $y = uvw$, $v \neq \epsilon$, and for all $x, z \in \Sigma^*$ and $i \geq 0$,

$$xuz \in A \iff xuv^iz \in A.$$

45. Let A be any subset of $\{a\}^*$ whatsoever.

*H(a) Show that A^* is regular.

**S(b) Show that

$$A^* = \{a^{np} \mid n \geq 0\} - G,$$

where G is some finite set and p is the greatest common divisor of all elements of the set $\{m \mid a^m \in A\}$. This is a generalization of the so-called postage stamp problem: any amount of postage over 7 cents can be made with some combination of 3 and 5 cent stamps.

46. Prove that the DFA with 15 states shown in Lecture 5 for the set (5.1) is minimal.

47. Minimize the following DFAs. Indicate clearly which equivalence class corresponds to each state of the new automaton.

(a)

		a	b
→	1	6	3
	2	5	6
	$3F$	4	5
	$4F$	3	2
	5	2	1
	6	1	4

(b)

		a	b
→	1	2	3
	2	5	6
	$3F$	1	4
	$4F$	6	3
	5	2	1
	6	5	4

(c)

		a	b
→	$0F$	3	2
	$1F$	3	5
	2	2	6
	3	2	1
	4	5	4
	5	5	3
	6	5	0

(d)

		a	b
→	0	3	5
	1	2	4
	2	6	3
	3	6	6
	$4F$	0	2
	$5F$	1	6
	6	2	6