Homework 4

1. Show that the following sets are not regular.

   (a) $\{a^n b^m \mid n = 2m\}$

   (b) $\{x \in \{a, b, c\}^* \mid x \text{ is a palindrome; i.e., } x = \mathbf{rev}(x)\}$

   (c) $\{x \in \{a, b, c\}^* \mid \text{the length of } x \text{ is a square}\}$

   (d) The set PAREN of balanced strings of parentheses `( )`. For example, the string `((()())())` is in PAREN, but the string `)(()` is not.

2. The operation of *shuffle* is important in the theory of concurrent systems. If $x, y \in \Sigma^*$, we write $x \parallel y$ for the set of all strings that can be obtained by shuffling strings x and y together like a deck of cards; for example,

$$ab \parallel cd = \{abcd, acbd, acdb, cabd, cadb, cdab\}.$$

The set $x \parallel y$ can be defined formally by induction:

$$\epsilon \parallel y \stackrel{\text{def}}{=} \{y\},$$
$$x \parallel \epsilon \stackrel{\text{def}}{=} \{x\},$$
$$xa \parallel yb \stackrel{\text{def}}{=} (x \parallel yb) \cdot \{a\} \cup (xa \parallel y) \cdot \{b\}.$$

The shuffle of two languages A and B, denoted $A \parallel B$, is the set of all strings obtained by shuffling a string from A with a string from B:

$$A \parallel B \stackrel{\text{def}}{=} \bigcup_{\substack{x \in A \\ y \in B}} x \parallel y.$$

For example,

$$\{ab\} \parallel \{cd, e\} = \{abe, aeb, eab, abcd, acbd, acdb, cabd, cadb, cdab\}.$$

   (a) What is $(01)^* \parallel (10)^*$?

   (b) Show that if A and B are regular sets, then so is $A \parallel B$. (*Hint:* Put a pebble on a machine for A and one on a machine for B. Guess nondeterministically which pebble to move. Accept if both pebbles occupy accept states.)