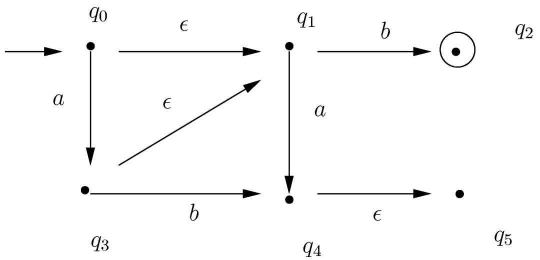
A partir de un AFN M con ϵ-transiciones se puede definir un AFN M' sin ϵ-transiciones tal que $L(M') = L(M)$.

Ejemplo

Sea M el siguiente



sea Δ su relación de transición. Vamos a definir un M' con relación de transición Δ': los estados siguientes:

- $\Delta'(q_0, a) = \{q_1, q_3, q_4, q_5\}$
- $\Delta'(q_0, b) = (\epsilon - c)(d((\epsilon - c)(q_0), b))$; donde

  $$(\epsilon - c)(q_0) = \{q_1\}, \quad d(\{q_1\}, b) = \{q_2\}, \quad (\epsilon - c)(q_2) = \{q_2\}$$

  por lo que

  $$\Delta'(q_0, b) = \{q_2\}$$
- $\Delta'(q_1, a)$:

  $$(\epsilon - c)(q_1) = \{q_1\}$$
  $$d(q_1, a) = \{q_4\}$$
  $$(\epsilon - c)(q_4) = \{q_4, q_5\}$$

  $$\Delta'(q_1, a) = \{q_4, q_5\}$$

- $\Delta'(q_1, b)$:

$$(\epsilon - c)(q_1) = \{q_1\}$$
$$d(q_1, b) = \{q_2\}$$
$$(\epsilon - c)\{q_2\} = \{q_2\}$$

$$\Delta'(q_1, b) = \{q_2\}$$

- $\Delta'(q_2, a)$:

$$(\epsilon - c)(q_2) = \{q_2\}$$
$$d(q_2, a) = \varnothing$$
$$(\epsilon - c)\varnothing = \varnothing$$

$$\Delta'(q_2, a) = \varnothing.$$

- $\Delta'(q_2, b) = \varnothing$.
- $\Delta'(q_3, a)$:

$$(\epsilon - c)(q_3) = \{q_3, q_1\}$$
$$d(\{q_3, q_1\}, a) = d(q_3, a) \cup d(q_1, a) = \varnothing \cup \{q_4\} = \{q_4\}$$
$$(\epsilon - c)(q_4) = \{q_4, q_5\}$$

$$\Delta'(q_3, a) = \{q_4, q_5\}$$

- $\Delta'(q_3, b)$:

$$(\epsilon - c)(q_3) = \{q_3, q_1\}$$
$$d(\{q_3, q_1\}, b) = d(q_3, b) \cup d(q_1, b) = \{q_4\} \cup \{q_2\} = \{q_4, q_2\}$$
$$(\epsilon - c)\{q_4, q_2\} = (\epsilon - c)(q_4) \cup \underbrace{(\epsilon - c)(q_2)}_{\varnothing} = \{q_4, q_5\}$$

$$\Delta'(q_3, b) = \{q_4, q_5\}$$

- $\Delta'(q_4, a)$:

$$(\epsilon - c)(q_4) = \{q_4, q_5\}$$
$$d(q_4, a) \cup d(q_5, a) = \emptyset$$

$$\Delta'(q_4, a) = \emptyset.$$

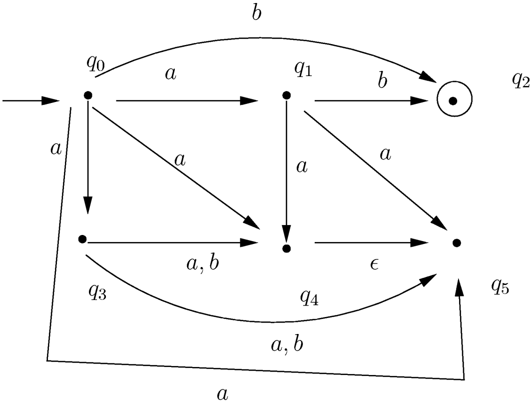
- $\Delta'(q_4, b)$:

$$(\epsilon - c)(q_4) = \{q_4, q_5\}$$
$$d(q_4, b) \cup d(q_5, b) = \emptyset$$

$$\Delta'(q_4, b) = \emptyset$$

- $\Delta'(q_5, b) = \Delta'(q_5, a) = \emptyset$.

Obtenemos que M' es



Nótese que

$$L(M) = \{b, ab\} = L(M').$$

Teorema

Sea

$$M = (Q, \Sigma, s, F, \Delta)$$

un AFN con ϵ*-transiciones. Entonces existe*

$$M' = (Q', \Sigma', s', F', \Delta')$$

un AFN sin ϵ*-transiciones tal que*

$$L(M') = L(M).$$

Dem.

Se definen

$$Q' = Q, \quad \Sigma' = \Sigma, s' = s$$

$$F' = \{q \in Q \,|\, (\epsilon - c)(q) \cap F \neq \varnothing\}$$

y si $q \in Q$ y $\sigma \in \Sigma$ entonces

$$\Delta'(q, \sigma) = (\epsilon - c)\big(d((\epsilon - c)(q), \sigma)\big)$$

Tenemos que demostrar que

$$w \in L(M') \Leftrightarrow w \in L(M).$$

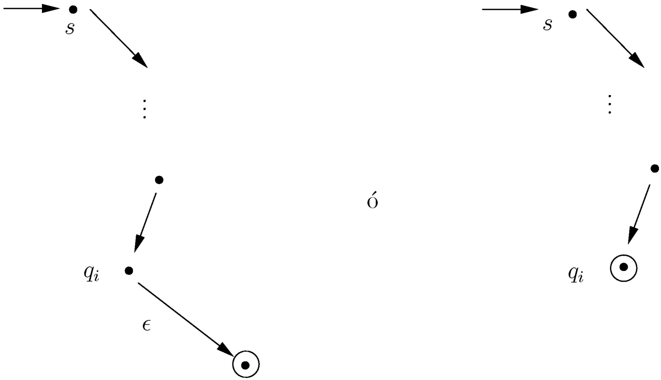
Si $w \in L(M')$ entonces $\Delta'(s', w) \cap F' \neq \varnothing$, esto es

$$\Delta'(s, w) \cap \{q \,|\, (\epsilon - c)(q) \cap F \neq \varnothing\} \neq \varnothing$$

por lo que existe $q_i \in Q$ tal que

$$q_i \in \Delta'(s, w) \text{ y } (\epsilon - c)(q_i) \cap F \neq \varnothing$$

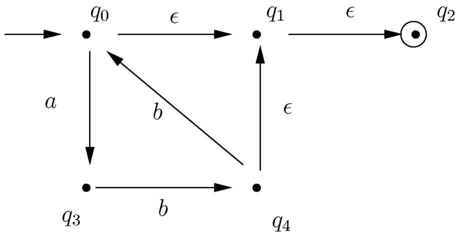
lo segundo indica que $q_i \in F$ o a q_i le sigue un estado final después de una o más ϵ-transiciones:



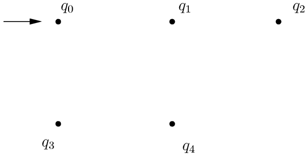
Lo que implica que $w \in L(M)$. El recíproco es similar.

Ejemplo

Sea



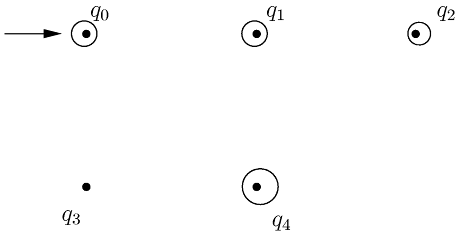
podemos encontrar M' un AFN sin ϵ-transiciones equivalente a M: por construcción, los estados de M' son los mismos que los de M:



los estados finales de M' son $F' = \{q \,|\, (\epsilon - c)(q) \cap F \neq \varnothing\}$:

q	$(\epsilon - c)(q)$	$(\epsilon - c)(q) \cap F \neq \varnothing$
q_0	$\{q_0, q_1, q_2\}$	$\{q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\varnothing$
q_4	$\{q_4, q_1, q_2\}$	$\{q_2\}$

de donde $F' = \{q_0, q_1, q_2, q_4\}$. Actualizamos nuestro diagrama de transiciones:



Ahora, recordemos que

$$\Delta'(q,\sigma) = (\epsilon - c)(d((\epsilon - c)(q),\sigma))$$

- $\Delta'(q_0, a)$:

$$(\epsilon - c)(q) = \{q_0, q_1, q_2\}$$

$$d(\{q_0, q_1, q_2\}, a) = d(q_0, a) \cup d(q_1, a) \cup d(q_2, a) = \{q_3\} \cup \varnothing\varnothing = \{q$$

$$(\epsilon - c)(q_3) = \{q_3\}$$

$$\Delta'(q_0, a) = \{q_3\}$$

- $\Delta'(q_0, b)$:

$$(\epsilon - c)(q_0) = \{q_0, q_1, q_2\}$$
$$d(\{q_0, q_1, q_2\}, b) = \varnothing \cup \varnothing\varnothing = \varnothing,$$

$$\Delta'(q_0, b) = \varnothing.$$

- $\Delta'(q_1, a)$:

$$(\epsilon - c)(q_1) = \{q_1, q_2\}$$
$$d(\{q_1, q_2\}, a) = d(q_1, a) \cup d(q_2, a) = \varnothing \cup \varnothing = \varnothing,$$

$$\Delta'(q_1, a) = \varnothing.$$

- $\Delta'(q_1, b) = \varnothing.$
- $\Delta'(q_2, a) = \varnothing.$
- $\Delta'(q_2, b) = \varnothing.$

- $\Delta'(q_3, a)$:

$$(\epsilon - c)(q_3) = \{q_3\}$$
$$d(q_3, a) = \varnothing$$
$$\Delta'(q_3, a) = \varnothing.$$

- $\Delta'(q_3, b)$:

$$d(q_3, b) = \{q_4\}$$
$$(\epsilon - c)(q_4) = \{q_4, q_1, q_2\}$$

$$\Delta'(q_3, b) = \{q_4, q_1, q_2\}.$$

- $\Delta'(q_4, a)$:

$$(\epsilon - c)(q_4) = \{q_4, q_1, q_2\}$$
$$d(\{q_4, q_1, q_2\}, a) = d(q_4, a) \cup d(q_1, a) \cup d(q_2, a) = \varnothing \cup \varnothing \cup \varnothing$$

$$\Delta'(q_4, a) = \varnothing.$$

- $\Delta'(q_4, b)$:

$$d(\{q_4, q_1, q_2\}, b) = d(q_4, b) \cup d(q_1, b) \cup d(q_2, b) = \{q_0\}$$
$$(\epsilon - c)(q_0) = \{q_0, q_1, q_2\}$$

$$\Delta'(q_4, b) = \{q_0, q_1, q_2\}.$$

Hemos obtenido M':